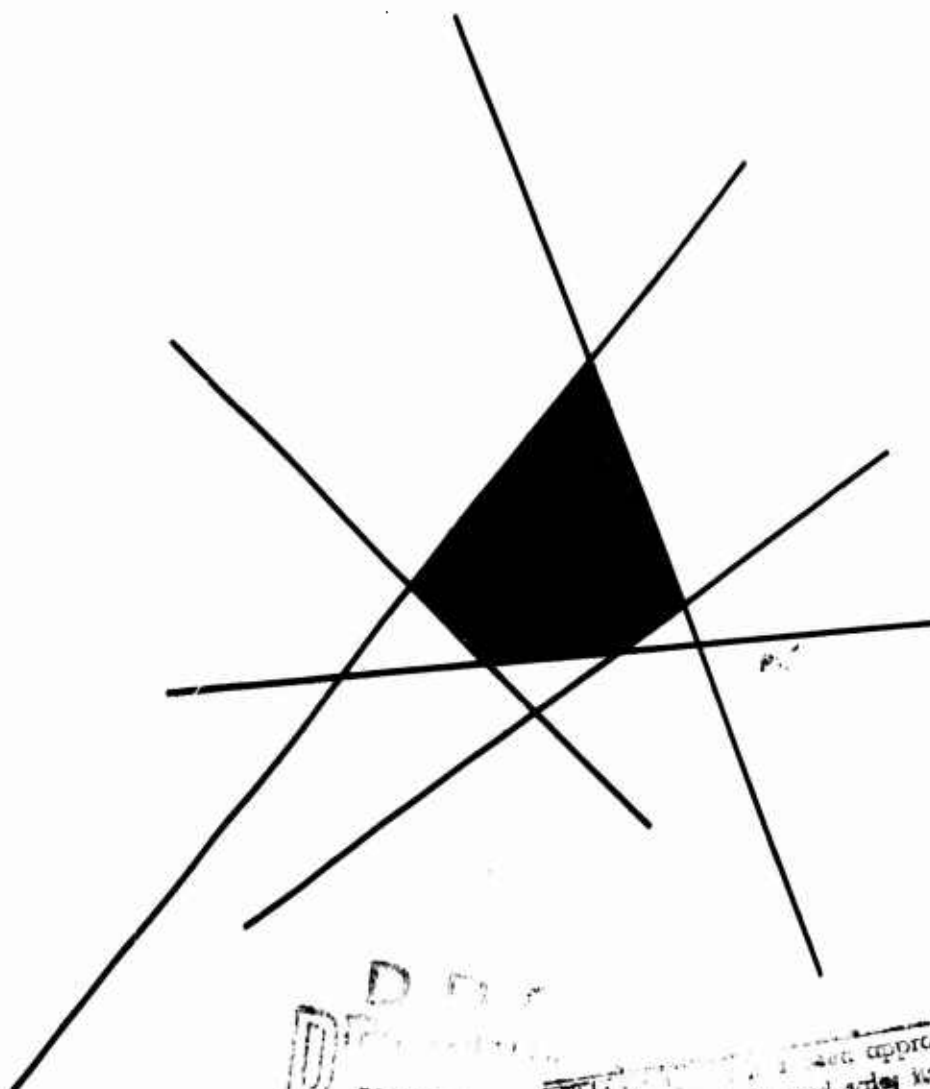


# A SHUTTLE CAR ASSIGNMENT PROBLEM IN THE MINING INDUSTRY

by  
GARY H. REYNOLDS

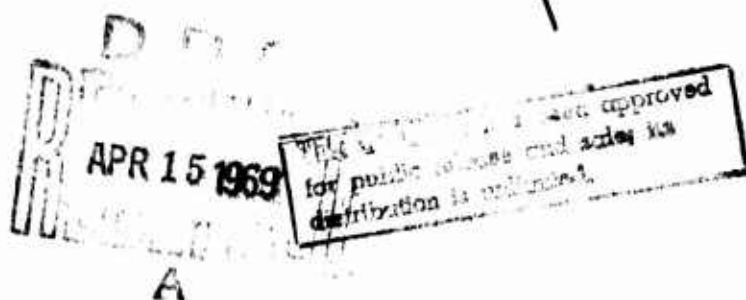
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IN THE MINING INDUSTRY<sup>†</sup>

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<sup>†</sup>Part of this paper went into the making of a term paper while a student in the Mathematics Department of the Pennsylvania State University. In this connection the author would like to thank Professor H. S. Hahn. The author also appreciates the time and experience made available to him as a Research Assistant in the Mining Department of the Pennsylvania State University. Finally, part of this research has been supported by the Office of Naval Research under Contract Nonr-222(83) and the National Science Foundation under Grant GP-8695 with the University of California. Reproduction in whole or in part is permitted for any purpose of the United States Government.

### ABSTRACT

Shuttle cars form a part of the standard equipment employed by the mining industry in the transportation of coal when using the room-and-pillar method for extracting coal from underground seams. These cars are subject to frequent failure, so the problem of assigning shuttle cars to sections of the mine is considered in this paper, with the objective of maximizing expected output. A particularly simple solution is found that could be used readily by any mine foreman.

## A SHUTTLE CAR ASSIGNMENT PROBLEM IN THE MINING INDUSTRY

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A preponderant percentage of the coal mining organizations in Pennsylvania employ the room-and-pillar method for extracting coal from underground seams. Using the room-and-pillar method a typical sequence of operations from coal face to surface would appear schematically as in Figure 1.

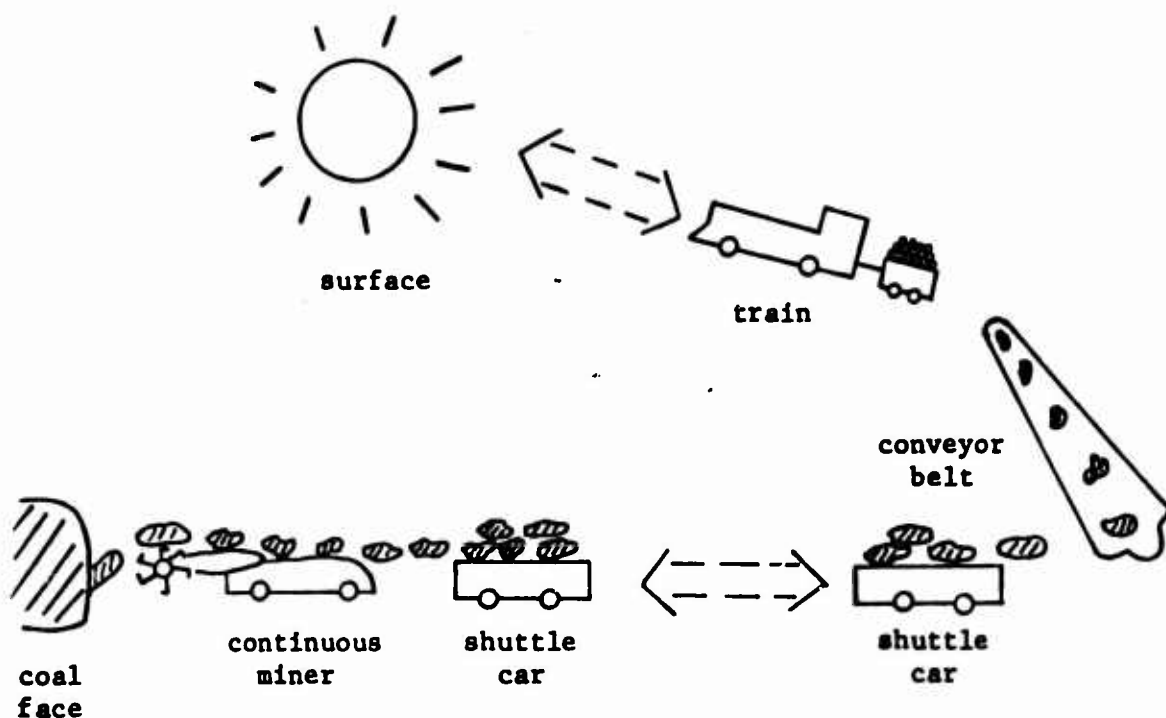


FIGURE 1

Barring unusual circumstances, every continuous miner has assigned to it two shuttle cars that make periodic trips from the continuous miner to the conveyor belt. Figure 1 shows only one section of a coal mine and similar operations are being carried on in many locations simultaneously.

Restricting our attention to one section of the mine, its effectiveness (based on the amount of coal per unit time being extracted) can be roughly estimated as follows:

- (a) If all components (continuous miner, shuttle cars, conveyor belt and train) are in operating condition the effectiveness of the section is 100%.
- (b) If all components except for one shuttle car are in operating condition, the effectiveness of the section is 67%.
- (c) If both shuttle cars are inoperative, the section effectiveness is 0%.

In (b), the 67% figure is a rough estimate. In the sequel, this figure is *essential* only in so far as the section effectiveness is *above* 50%. In general, this will be true. Under the conditions stated in (a), a shuttle car, having transferred its load to the conveyor belt, will *wait* in a byway while the other shuttle car is *still being loaded*. Thus, when one car becomes inoperative the remaining car *absorbs* the delay normally incurred in going from the conveyor belt to the continuous miner, so that the section effectiveness is above 50%.

Based on the foregoing observations, we can conclude that it is more desirable to have one shuttle car fail in each of two sections, rather than to have them both fail in one section. More explicitly, we consider the following

Problem:

Given a probability of failure,  $p$ , for each shuttle car in the mine, find an assignment placing two shuttle cars in each section which maximizes total expected output.

An assessment of the exact probability of failure for a shuttle car would be difficult to determine, being an intricate function of its age, past environment and machine design. It follows, however, from the corollary proved in the next section, that an explicit evaluation of failure probabilities is not necessary. A simple ranking of the shuttle cars in increasing reliability is sufficient to determine an optimal assignment.

Solution:

Given a mine with  $n$  sections, let the shuttle cars  $c_1, c_2, \dots, c_n, c_{n+1}, c_{n+2}, \dots, c_{2n}$ , be arranged by increasing order of reliability. That is,  $c_1$  denotes the least reliable machine and  $c_{2n}$  denotes the most reliable machine. Then the maximizing solution is given by the pairs  $\{c_1, c_{2n}\}, \{c_2, c_{2n-1}\}, \dots, \{c_n, c_{n+1}\}$ .

Finally, although it is, undoubtedly, difficult to interchange shuttle cars between sections in a coal mine we can take advantage of those opportunities when two or more sections come into proximity. By making these local exchanges we will be led to the global optimum.

### The Mathematical Model

We assign a probability of failure,  $0 \leq p_1 < 1$  to each shuttle car in the mine. For the  $k$ th section, the probability of both shuttle cars failing is given by the product  $p_1^k p_2^k$ , assuming the probabilities are independent. If both shuttle cars in the  $k$ th section are operating, the section effectiveness is 1; if only one car is operating its effectiveness is  $\delta$ , where  $1 \geq \delta > 0.5$ ; otherwise the section effectiveness is 0. For a mine composed of  $n$  sections, we maximize the total expected output as a function of the sequential ordering of  $S = [p_1^1, p_1^2, \dots, p_1^n, p_2^1, p_2^2, \dots, p_2^n]$ . Thus, the total expected output can be written

$$f(S) = \sum_{k=1}^n \left[ 1 - p_1^k p_2^k - p_1^k (1 - p_2^k) - p_2^k (1 - p_1^k) \right] + \delta \sum_{k=1}^n \left[ 1 - p_1^k p_2^k - (1 - p_1^k)(1 - p_2^k) \right]$$

or,

$$f(S) = \left[ n + (\delta - 1) \sum_{k=1}^n (p_1^k + p_2^k) \right] - (2\delta - 1) \sum p_1^k p_2^k. \quad (1)$$

The first term, in square brackets, remains invariant under any sequential ordering. Since  $\delta > 0.5$ , maximizing Equation 1 is equivalent to minimizing

$$g(S) = \sum p_1^k p_2^k. \quad (2)$$

If we restricted the problem to minimizing  $g$  over the two sequences  $A = [p_1^1, p_1^2, \dots, p_1^n]$  and  $B = [p_2^1, p_2^2, \dots, p_2^n]$ , not allowing interchanges between them, a simple solution is found as a part of the folklore of mathematics (e.g., see Hardy, Littlewood and Polya [2]). In this case a value of  $g$  is minimum if, and only if,

$$p_1^k < p_1^l \text{ implies } p_2^k \geq p_2^l, \text{ for } 1 \leq k, l \leq n. \quad (3)$$

To solve the problem when interchanges between the two sequences are allowed we use the set theoretic notion of "systems of common representatives." By a *partition* of a set  $T$  into  $n$  components is meant a collection of  $n$  disjoint subsets,  $T_1, T_2, \dots, T_n$ , of  $T$  satisfying

$$T = T_1 \cup T_2 \cup \dots \cup T_n.$$

Two partitions

$$T = T_1 \cup T_2 \cup \dots \cup T_n$$

and

$$T = T'_1 \cup T'_2 \cup \dots \cup T'_n \quad (4)$$

of  $T$  into  $n$  components are said to have a *system of common representatives*, SCR, if there exists a subset,  $R$ , of  $n$  elements of  $T$  such that  $R \cap T_i \neq \emptyset$  and  $R \cap T'_i \neq \emptyset$  for each  $i = 1, 2, \dots, n$ . Two partitions, (4), of  $T$  always have an SCR if each of the subsets  $T_i, T'_i, i = 1, 2, \dots, n$ , contain the same number of elements of  $T$ . This important theorem on the existence of an SCR was first proved by König [3] in terms of bipartite graphs. P. Hall [1] generalized König's theorem and established his results in the context of set representatives.

We are now in a position to prove the main result. First, however, we simplify the notation. Let  $S = [p_1^1, p_1^2, \dots, p_1^n, p_2^1, p_2^2, \dots, p_2^n] = [d_1, d_2, \dots, d_n, d_{n+1}, d_{n+2}, \dots, d_{2n}]$  and call two elements  $d$  and  $\bar{d}$  of  $S$  *conjugate* if  $d = d_i$  and  $\bar{d} = d_{i+n}$ , or  $d = d_{i+n}$  and  $\bar{d} = d_i$ , for some  $i = 1, 2, \dots, n$ . Let  $p(S)$  denote the set of all arrangements of  $S$ .



Theorem:

Let  $S^* = [d_1^*, d_2^*, \dots, d_{2n}^*]$ . Then  $g(S^*) \leq g(S)$  for each  $S \in p(S)$  if, and only if, the following proposition is true: If  $d_i^*$  and  $d_j^*$  are any two elements of  $S^*$  not conjugate to each other, then  $d_i^* < d_j^*$  implies  $\bar{d}_i^* \geq \bar{d}_j^*$ .

Proof:

Suppose  $d_i^*$  and  $d_j^*$  are not conjugate in the optimal solution  $S^*$  and that  $d_i^* < d_j^*$ . Let  $p_1^k = d_i^*$ ,  $p_1^l = d_j^*$ ,  $p_2^k = \bar{d}_i^*$  and  $p_2^l = \bar{d}_j^*$ . From (3) we see that  $\bar{d}_i^* \geq \bar{d}_j^*$ .

On the other hand, let  $S^*$  satisfy the proposition and let  $S = [d_1, d_2, \dots, d_{2n}]$  by any arrangement in  $p(S)$ . Consider the  $2n$  components of  $S^*$  and  $S$  as distinct elements of a set and form the subsets

$$S_k^* = [d_k^*, d_{n+k}^*]$$

and

$$S_k = [d_k, d_{n+k}], \quad k = 1, 2, \dots, n.$$

Then

$$T = S_1^* \cup S_2^* \cup \dots \cup S_n^*$$

and

$$T = S_1 \cup S_2 \cup \dots \cup S_n$$

form two partitions of the set  $T$ . Since each of the subsets  $S_k^*$  and  $S_k$ ,  $k = 1, 2, \dots, n$ , consist of exactly two elements of  $T$  it follows that there exists an SCR. So assume the subsets of the two partitions have been renumbered, if necessary, so that  $S_k^* \cap S_k \neq \emptyset$ , for  $k = 1, 2, \dots, n$ . Choose one element,  $a_k$ , from each of the sets  $S_k^* \cap S_k$  to form the SCR,

$$R = [a_1, a_2, \dots, a_n] . \quad (5)$$

Define

$$\{b_k^*\} = S_k^* - \{a_k\}$$

and

$$\{b_k\} = S_k - \{a_k\} , k = 1, 2, \dots, n .$$

We then have that

$$g(S^*) = \sum_{k=1}^n a_k b_k^*$$

and

$$g(S) = \sum_{k=1}^n a_k b_k .$$

We can further assume that the sequence  $[a_1, a_2, \dots, a_n]$  is monotonically increasing. Thus, if  $a_i < a_j$ , from (5) we see that  $S_i^* \neq S_j^*$ , which implies that  $a_i$  and  $a_j$  are not conjugate with respect to  $S^*$ . Since  $S^*$  satisfies the proposition we can deduce  $b_i \geq b_j$ . Hence from (3),

$$g(S^*) \leq g(S) .$$

Q.E.D.

As an immediate consequence of the theorem we have the following

Corollary:

Let  $S^* = [d_1^*, d_2^*, \dots, d_n^*, d_{2n}^*, d_{2n-1}^*, \dots, d_{n+1}^*]$ . If  $[d_1^*, d_2^*, \dots, d_n^*, d_{n+1}^*, d_{n+2}^*, \dots, d_{2n}^*]$  is monotonically increasing, then

$$g(S^*) \leq g(S)$$

for each  $S \in p(S)$  .

For if  $d_1^* < d_j^*$  and  $d_1^*$  and  $d_j^*$  are not conjugate then

$$\bar{d}_1^* = d_{2n-1+1}^* \geq d_{2n-j+1}^* = \bar{d}_j^* .$$

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